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Symmetries in N=4 Supergravities

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Chapter 5

Conclusions, Discussion and Developments

In this chapter we summarize and discuss the important conclusions and try to give directions for future developments. In particular we try to answer the questions posed in the introduction. As to be expected from a thesis about a small piece of a big jigsaw puzzle, there is not a single clear conclusion. Therefore we gather the main results that are presented in the thesis.

We have shown how the degrees of freedom that result from a group manifold reduction of a six-form gauge field can be analyzed using the cohomology of Chevalley and Eilenberg. No degrees of freedom are lost since a group manifold has Euler characteristic zero. In the same analysis we showed that all compact connected Lie groups of dimension smaller than seven are products of $SO(3)$, $SU(2)$ and $U(1)$.

We have shown a technique to find the global symmetry group of a dimensionally reduced ten-dimensional supergravity. The technique is an extension of the work of Pope and Lü [114]. Although the result is generally known, the technique is new and explains how the coset structure of the scalars arises when one goes down in dimensions. We have however not investigated what happens when the higher-dimensional theory already has scalars that parameterize a coset.

We have found no stable vacuum for semisimple gaugings of $\mathcal{N} = 4$ supergravity coupled to 6 vector multiplets at the identity point of the scalar manifold $SO(6,6)/SO(6) \times SO(6)$. The identity point of a coset G/K is that point that is identified with the compact subgroup K of G. For many gaugings the identity point corresponds to an extremum of the scalar potential.

We have ignored one aspect in the semisimple gauging of $\mathcal{N} = 4$ supergravity coupled to 6 vector multiplets. There exists a two-parameter family of invariant metrics on $\mathfrak{so}(1,3)$. We leave it for future research to discuss whether this allows stable vacua at the identity point of $SO(6,6)$.

We have shown that the only the CSO-groups that can be a subgroup of the gauge group in $\mathcal{N}=4$ supergravity are $CSO(p,q,r)$ with $p+q+r=4$. We have shown in the same section that there is a straightforward way to investigate the CSO-gaugings by working out how the adjoint representation of a $\mathfrak{cso}$-algebra can be embedded in the vector representation of $\mathfrak{so}(6,6)$. It is worth remarking that the only CSO-groups and SO-groups that can be used for gauging $\mathcal{N}=8$ are the $CSO(p,q,r)$ with $p+q+r=8$ [170–172, 178–184].

The CSO-groups are just one family of nonsemisimple groups. To the knowledge of the author there is no special reason to gauge CSO-groups or semisimple groups in $\mathcal{N}=4$ supergravity. Hence gauging a group in $\mathcal{N}=4$ is always ad hoc; why not another group? Since the number of coupled vector multiplets is arbitrary, the dimension of the gauged group is arbitrary. This makes the search for stable de Sitter vacua in $\mathcal{N}=4$ supergravities by gauging different groups a bit like a lottery.

The above mentioned conclusions are the main conclusions that we wish the reader who wants to remember something, remembers. We now turn to the questions posed in the Introduction. We give the answers, if any are found, in the same order as the questions are listed.

-1- The role of the $SU(1,1)$-angles is still obscure. Their presence breaks a few symmetries. Firstly, in the ungauged supergravity the global symmetry $SO(6,n)$ is broken. Secondly, in the gauged theory the symmetry group of the potential is broken from an $O(6)$ to an $SO(6)$ in presence of the $SU(1,1)$-angles. We have not obtained the $SU(1,1)$-angles by a toroidal or group manifold dimensional reduction. Although the $SU(1,1)$-angles can be seen as a subset of the full set of parameters that determine a gauging of $\mathcal{N}=4$ supergravity [146], this does not fix a higher-dimensional origin.

-2- A Lie group G can be used to gauge $\mathcal{N}=4$ supergravity if and only if its Lie algebra $\mathfrak{g}$ admits an invariant metric with n_+ positive eigenvalues and n_- negative eigenvalues such that $\min(\mathrm{n}_-,\mathrm{n}_+)\leq 6$.

-3- The answer is already known in the literature, see [185], and can be summarized as follows. If the scalars in a supergravity parameterize a coset G/K, the kinetic term is determined by the invariant metric on G/K. The invariant metric is invariant and Riemannian only if K is the maximal compact subgroup of G. Note that we cannot divide out the noncompact subgroup since the noncompact part is not a subgroup. Hence the only coset G/K that does not give rise to ghosts for the scalars is the coset where K is the maximal compact subgroup of G.

-4- If no fluxes are present the isometry group of the internal manifold of a dimensional reduction is a subgroup of the global symmetry group. This can be seen by performing a dimensional reduction of the isometries acting on the fields. The gauge symmetries that are present in the higher-dimensional theory can give rise to an enlarged symmetry group. For example, when a theory with a two-form gauge field present is reduced over $SU(2)$ the symmetry group is enlarged from $SU(2)$ to $CSO(3,0,1)$.

When fluxes are taken into account, the full isometry group is in general no longer a symmetry of the theory. In the general case the symmetry group is smaller. A dimensional reduction with fluxes gives rise to a gauged supergravity, in which the global symmetry group of the ungauged theory is broken to a smaller local symmetry group.

-5- The last question is the hardest to answer. The fate of string/supergravity theories is simply not known. The richness of the theories is so large that many more years are needed to get an overview of the interplay between string theories and supergravity theories and their solutions.

In the year 2007 the LHC will start looking for supersymmetry. If supersymmetry is found, this will be a great triumph for string/supergravity theory and will motivate more intensive research. If supersymmetry is not found, this still does not mean supersymmetry does not exist since the energies above which supersymmetry has to be seen, depend strongly on the models and scenarios that are used. However, if supersymmetry is not found, superstrings and supergravities will become a more outback area of physics, isolated from the other physical disciplines. It does not mean that string/supergravity theories are useless, to the contrary, we have learned much (theoretical) physics and mathematics while investigating them.

We wish to conclude this final chapter by posing some questions for future research:

-a- Can dimensional reductions that are not Kaluza–Klein reductions, such as orbifold reductions, give rise to parameters in a lower-dimensional theory that resemble the $SU(1,1)$-angles?

-b- If one performs a toroidal reduction of a theory that has coset scalars, what coset do the lower-dimensional scalars parameterize?

-c- Can statements be made about global properties of the scalar potential of $\mathcal{N}=4$ supergravity coupled to n vector multiplets, such as the existence of global or local minima?

-d- Can the potential of $\mathcal{N}=4$ supergravity drive the present-day acceleration of the universe?

-e- Can general solutions to the parameters $\xi_{\alpha M}$ and $f_{\alpha KLM}$ of [146] that determine the most general gauging of $\mathcal{N}=4$ supergravity be found?

Some of these question may find an answer easily, others might never be answered. And there are many other questions of course, but as always in science, posing the right questions is harder than giving an answer to a given question.